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The Spectrum of Radiocarbon [and Discussion]

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The spectrum of radiocarbon

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The power spectral density of the specific activity of radiocarbon variations, using an absolute chronology based on tree ring count, exhibits spectral lines at a number of periods: 2300, 964, 753, 717, 493, 413, 357, 229 and 208 years as well as recording a secular variation over the full 9000-year record. These variations appear in both the La Jolla and Belfast radiocarbon records and some are also detected in the Camp Century ^{10}Be record, though its secular variation appears to lead that of the radiocarbon by about 2100 years. Of the total number of nine prominent radiocarbon features, most are mutually dependent with perhaps only three independent lines. The 208-year period appears modulated by the long 2300-year period. The evidence of modulation of the 208- (and possibly 229-) year period(s) by the 2300-year period suggests a solar source for the latter features, through 200-year spectral features are also detected in the tree ring spectrum, thus tying Sun and climate together. A solar source signalled jointly through correlated radiocarbon and atmospheric temperature variations suggests that solar hydromagnetic and bolometric variations are coupled. Moreover, as evidence is lacking for variations in ^{10}Be or in the geomagnetic field with a period of 2300 years, by a process of elimination together with suggestive global ocean deep water return times of more or less a millennium, a surmise is that chemical resonances may account for the periodicity in radiocarbon with the variation in oceanic carbonate concentration recorded in the radiocarbon record as tracer. The evidence for correlated oscillations in air temperature detected by tree-ring-growth cyclicity in Campito Mt bristlecone pine trees and the radiocarbon variability is consistent with a model of atmosphere–ocean resonances underlying the radiocarbon periodicities.

1. Introduction

That the atmospheric inventory of radiocarbon (^{14}C) is constant was long a basic principle of radiocarbon dating, though it was shown as early as 1958 by de Vries (1958) that at least for the short term of a few hundred years this principle is violated (see de Jong & Mook 1980). The well-known major and possibly secular variation of *ca.* 10% was also discovered subsequent to Ferguson's extension of the bristlecone pine chronology (Ferguson 1970; Suess 1965, 1967, 1978; Damon *et al.* 1972). As the principle of reservoir constancy has been slowly abandoned, the explanation of the time variability of the radiocarbon record has become a central problem of geophysics and geochemistry and possibly solar physics. Cosmic ray (CR) flux variations are hydromagnetic and probably involve the Sun though marginally likely to arise from interstellar variability (Sonett *et al.* 1987). Determination of the various forcing functions responsible for the variations in the radiocarbon record is a major problem in the study of carbon on Earth. It is this problem that we explore in this paper.

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Of the eight isotopes of carbon, radioactive (^{14}C) is produced terrestrially primarily by the nuclear reaction

$$^{14}N(n, p) \rightarrow {}^{14}C, \tag{1}$$

where ^{14}N is atmospheric. ^{14}C decays by

$$^{14}C \rightarrow {}^{14}N + \nu^{-} + \beta^{-}, \tag{2}$$

where ν^{-} is the antineutrino and β^{-} the electron. The half-life of radiocarbon, $\rho_{\frac{1}{2}} = 5730$ years (Lederer *et al.* 1967). The ultimate source of radiocarbon is traceable to the CR flux incident upon the atmosphere from which, by spallations, at atmospheric neutron sea is generated. It is these neutrons that participate in the N(n, p) reaction yielding ^{14}C (Lingenfelter & Ramaty 1970; O'Brien 1979). If the CR flux were constant, the atmospheric ^{14}C inventory would be in secular equilibrium.

Carbon is a geochemically active element, making it difficult to trace through the environment, but at the same time providing an important tracer for the physics and chemistry of the oceans, atmosphere and biosphere. Because of the reservoir capacity of the atmosphere and the relatively long half-life of ^{14}C, the atmosphere act like a low-pass filter with periods *ca.* 100 years attenuated in amplitude by a factor of *ca.* 20 (Houtermans *et al.* 1973; Siegenthaler *et al.* 1980). This tends to increase the difficulty of measurements involving short periods as the signal:noise ratio is small. The specific variable that occupies a central role is the $\Delta^{14}C$ or delta radiocarbon defined by $\Delta^{14}C = (^{14}C_{ref} - {}^{14}C_{inv})/{}^{14}C_{inv}$ where $^{14}C_{ref}$ is the chronological reference activity at time t determined from tree rings count and $^{14}C_{inv}$ is the true measured activity. It is customary to multiply this value by 10^3, giving units of 'per mille'. Because of the very complex chemistry of radiocarbon, the certification of spectral features can be materially aided by reference to the Camp Century ^{10}Be record (Beer *et al.* 1983) for which long-period variability is most likely assignable to modulation of the incident CR flux, interplanetary or geomagnetic or both.

The radiocarbon record is obtained almost without exception from measurement of radiocarbon ages in wood from trees for which an absolute chronology exists by virtue of growth ring count. Much of the variability is quasi-periodic, though the long trend (*ca.* 10000–12000 years), discussed next, may be secular. The records often though not exclusively considered by us are those of Suess at the University of California (La Jolla), the Belfast (Pearson) sequence (Pearson *et al.* 1986) and to a slightly lesser extent the sequence from Rhinegraben oak for which the chronology was worked out by Beker and the ^{14}C record by Suess.

Delta radiocarbon records are basically noisy. As putative signal levels are low, it is important to establish the primary statistical properties of these time sequences so that we know just what it is that we are dealing with and what the likelihood is of the appearance of artefacts. Some idea of the moments and of the stationarity is given by dividing the sequences into halves before computation of the moments. Both the La Jolla and Belfast subsequences are normally distributed; Belfast is strongly stationary whereas La Jolla shows non-stationarity at great age as a result of lessened accuracy. It becomes closely stationary when truncated to the length of the Belfast sequence (520 B.C.–1835 A.D.) (stationarity can be strong, weak, or absent depending upon independence of various moments upon time). To what degree the normal distribution is indicative of noise rather than signal has not been established.

2. The long trend

The 'secular' trend is the most obvious feature of the radiocarbon record, a 14% decrease in the atmospheric radiocarbon inventory from *ca.* 6000 B.C. to a broad minimum near 500 A.D. followed by a sharp rebound by about 2% to the modern value. The trend is demonstrated convincingly in figure 1, showing both the La Jolla and Belfast records low-pass filtered to and upper frequency of 0.01 a^{-1} (100 years). Detrending conventionally is by least squares fitting of period, amplitude and phase of a sinusoid (see, for example, Damon *et al.* 1988) and is attributed to modulation of the incident CR flux at the top of the atmosphere by changes in the geomagnetic field (Bucha 1969, 1970; Creer 1989; Barton *et al.* 1979) and by changes in the global carbon reservoir (Lal & Revelle 1984). In this paper we use the alternate of a sine wave and a third-order Legendre polynomial to detrend the record.

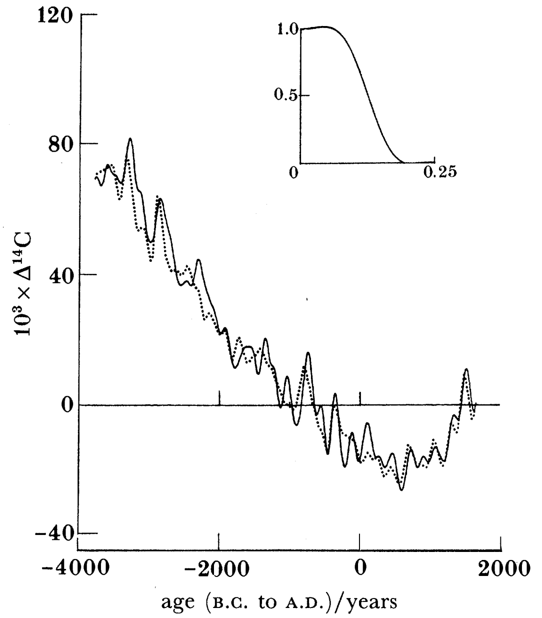

Figure 1. La Jolla (solid line) and Belfast (broken line) radiocarbon time sequences filtered to remove periods less than *ca.* 100 years. Inset shows filter amplitude transfer function. Periodicity of *ca.* 200 years appears prominently. (From Sonett (1985).)

By representing the long-term trend as a sinusoid of 8000-year period and using the archaeomagnetic data of Bucha (1970); Suess (1969, 1970) found an attenuation (the box model of Houtermans *et al.* (1973)) of 7 times and a phase shift of 30° between the atmospheric inventory and the radiocarbon production rate. To match the 0.52 power law for geomagnetic modulation of the radiocarbon production (Elsasser *et al.* 1956), the Creer archaeomagnetic data and the Belfast radiocarbon record suggest a phase lag of 500 years and an attenuation factor of 3.2 between production and inventory. The phase shift is consistent with that of Houtermans *et al.* but the attenuation is less. Thus a contribution from a non-magnetic perturbation is suggested. Lal & Revelle (1984) have shown that an increase in atmospheric pCO_2 could result in a decrease of about 7% in $\Delta^{14}C$, making a climatic effect possible.

The trend is also detected in the ^{10}Be record from the Greenland Camp Century ice core

(Beer *et al.* 1988) that is used to serve as a chemistry-free proxy for ^{14}C production. (The atmospheric beryllium inventory follows the production rate closely with little lag (Raisbeck & Yiou 1981).) But the beryllium trend is somewhat difficult to see because of the lack of reservoir-induced high-frequency attenuation and the trend is somewhat obscured by short-wavelength phenomena. A puzzling result from comparing the ^{10}Be and Belfast radiocarbon records is a peak in their correlation for a lag of 2100 years in the ^{14}C record against ^{10}Be. If real, we have no explanation for this time shift, but it may be a computational artefact because of high-frequency oscillations in the ^{10}Be record corrupting the Legendre function fit.

3. Power spectral density

The spectral interval considered here is restricted to periods greater than *ca.* 200 years. Suess (1980) reported previously unpublished calculations of Kruse of the power spectrum of the La Jolla $\Delta^{14}C$ record showing spectral lines at 2400, 930, 498, 308, 202, and lesser years. Neftel *et al.* (1981) confirmed the 200-year period in the La Jolla $\Delta^{14}C$ spectrum. A more-detailed statistical analysis (Sonett 1984) showed marginal evidence that the *ca.* 200 year period was subject to modulation by a longer period at *ca.* 2000 years. Finney & Sonett (1988) obtained the spectrum of the Belfast radiocarbon record using the new bayesian algorithm (Bretthorst 1988) and Finney (1988) has applied it to both the La Jolla and newer Belfast sequences and shows a high degree of compatibility of the two records and the statistical reliability of the spectral features.

The $\Delta^{14}C$ spectrum computed by using the discrete Fourier transform (DFT) (figure 2*a*) and maximum entropy (MEM) (figure 2*b*) confirms the complexity indicated earlier by Suess; some 10 spectral lines of differing intensity in the range of 200–2300 years are present. Both the La

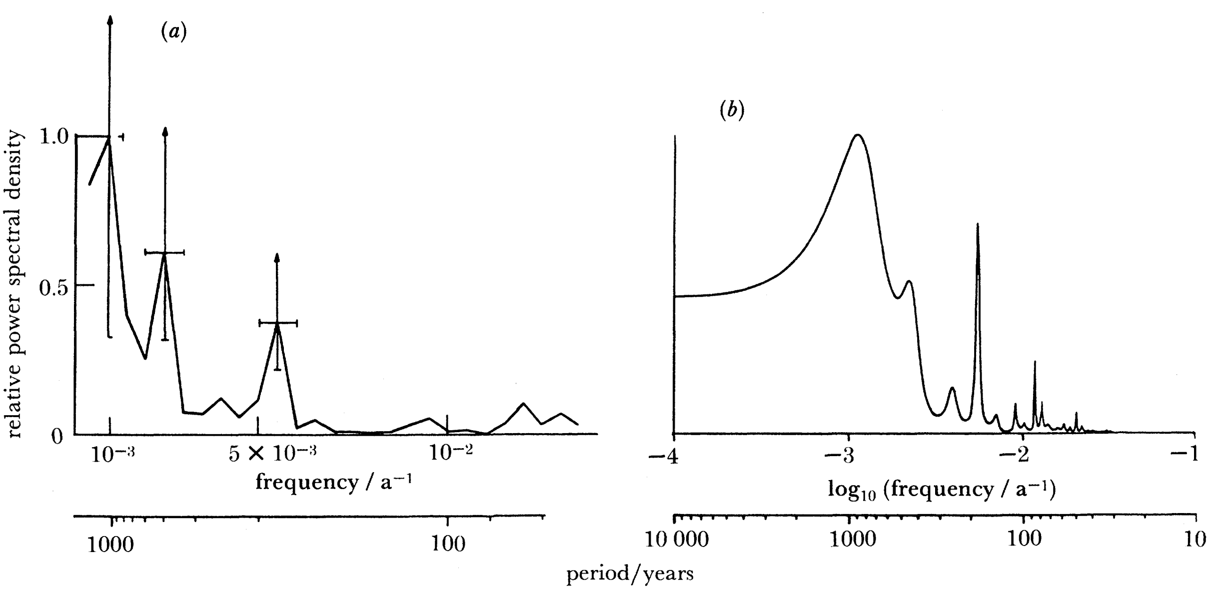

Figure 2. (*a*) Discrete power spectral density (PSD) of La Jolla ^{14}C sequence. Vertical bars are χ^2 amplitude estimates. Horizontal bars are estimated frequency resolution based upon computation steps, i.e. $f_0, f_1, \ldots$, where f_0 is the base frequency. (*b*) Maximum entropy PSD of La Jolla sequence showing prominent 200-year period. Resolution of both spectra are just sufficient to show major very-long period power, lesser at intermediate periods, and significant power in 200-year neighbourhood.

Jolla and Belfast records show similar lines. With decreasing period, spectral features are increasingly damped because of the atmospheric reservoir capacity. Most power appears at long periods. But there frequency resolution is reduced, and computational points may not be centred on line peaks leading to errors in estimation of amplitude. Moreover, line-frequency estimates are biased by window convolution with the data leading to side lobe and leakage problem. Some relief from the convolutional problem occurs by using the MEM algorithm, but this is counteracted by the lack of an adequate theory for specifying the optimum computational order. Nevertheless, MEM is a useful tool when used judiciously.

In the algorithm for computation of spectra developed by Jaynes and by Bretthorst (1987) one or more model functions are fitted to the data; the choice of an orthogonal coordinate basis set into which the data are projected eliminates side lobes. Additionally, data interpolation is unnecessary. Model probability is computed by projection of the data vector (in a Hilbert space) onto orthogonalized sinusoidal model functions. These provide a model likelihood. Table 1 gives model estimates using the combined La Jolla and Belfast records. Because the Bretthorst algorithm yields frequency probability estimates for a prior model (sines and cosines here) a spectrum is not directly provided. However, a spectrum can be constructed (figure 3) by plotting lines centred on the frequency probability maxima and with widths given by $1/e$ of the maximum probability. Then line amplitudes are made to correspond to model amplitudes.

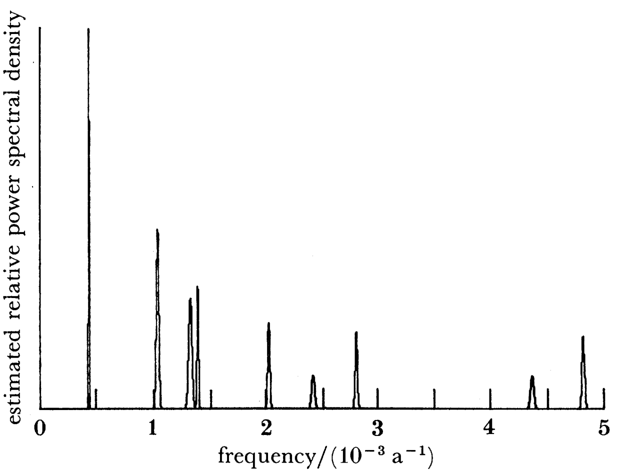

FIGURE 3. Spectral construction of spectrum from sine or cosine model bayesian estimates using both Belfast and La Jolla $\Delta^{14}C$ data. Line widths at base correspond to $1/e$ decrease in frequency positional probability from line centre. Amplitudes are estimated from sine and cosine model components.

Many of the lines appear to be linearly dependent upon one another. It is straightforward to show that all nine of the most prominent lines can be explained by linear combinations of a non-unique choice of three of the lines. As noted earlier we make use of the ^{14}Be record to distinguish the direct CR variability. Table 2 shows the frequencies and amplitudes of the beryllium record determined by the Bretthorst algorithm. However, the beryllium sample ages are based upon ice flow modelling and thus are intrinsically of greater uncertainty than indicated merely from the spectral modelling. Therefore, we do not report frequency uncertainties in table 2.

Lines $f_{Be,2}$ and $f_{Be,3}$ are within *ca.* 3% of $f_{C,5}$ and $f_{C,8}$ in frequency. Because this implies less

Table 1. Bayesian estimates of ^{14}C spectral lines

$10^4 \times$ frequency[a] / a^{-1}	period / years	amplitude	phase[b]
4.3267 ± 0.0891	2311	4.38	−119
1.0372 ± 0.0290	965	2.97	98
1.3276 ± 0.0408	753	2.34	−3
1.3941 ± 0.0186	717	2.46	76
2.0290 ± 0.0256	493	2.07	145
2.4209 ± 0.0357	413	1.27	87
2.8043 ± 0.0267	357	1.94	13
4.3637 ± 0.0400	229	1.28	−146
4.8170 ± 0.0304	208	1.90	129

[a] Errors are 2σ.
[b] Angles are measured + (counterclockwise) and − (clockwise).

Table 2. Camp Century ^{10}Be frequency and amplitude estimates

(Subscript c means cosine component; subscript s means sine component.)

line	frequency	amplitude
$f_{Be,1,c}$	3.0544×10^{-4}	-0.0923 ± 0.0155
$f_{Be,1,s}$		-0.0484 ± 0.0121
$f_{Be,2,c}$	1.9550×10^{-3}	0.0501 ± 0.0156
$f_{Be,2,s}$		0.0698 ± 0.0207
$f_{Be,3,c}$	4.478×10^{-3}	-0.0631 ± 0.0254
$f_{Be,3,s}$		-0.0352 ± 0.0231

than a full cycle difference between these lines in the two records, a possible common forcing mechanism is suggested. Contrasting this $f_{Be,1}$ and $f_{C,1}$ frequencies differ by 30%. Thus it is unlikely that forcing by the underlying non-dipole component of the geomagnetic field is responsible for these features. $f_{C,1}$ has been suggested as a result of geomagnetic modulation (Damon *et al.* 1990) but the parameter detected in the geomagnetic field at 2300 years is the direction sometimes attributed wholly to variations in the magnitude of the quadrupole moment. But assuming that the whole field participates in modulation, primarily the direction rather than the magnitude of the moment would be involved, and this cannot globally affect the CR flux.

The following assignment rationale assumes that the longest radiocarbon period $f_{c,1}$ is primary. Shuffling linear combinations of frequencies discloses that either $f_{C,2}$ or $f_{C,4}$ and $f_{C,8}$ or $f_{C,9}$ must be primary. Identification of $f_{C,5}$ as a second harmonic of $f_{C,2}$ and its appearance in the ^{10}Be record strengthens the supposition that $f_{C,2}$ is primary. It absence in the ^{10}Be may be because of its low amplitude. The appearance of $f_{C,8}$ in both the radiocarbon and ^{10}Be records suggests it to be primary (Finney 1988). But all these assignments should be regarded as tentative. In making the inferred relation to the ^{10}Be record note should be taken of the very restricted 141-datum ^{10}Be record. Table 3 shows line assignments satisfying these restrictions.

TABLE 3. A PLAUSIBLE ^{14}C LINE SOURCE ASSIGNMENT

symbol	period	source
$f_{C,1}$	2311	primary
$f_{C,2}$	965	primary
$f_{C,3}$	753	$3f_{C,1}$
$f_{C,4}$	717	$f_{C,2}+f_{C,1}$
$f_{C,5}$	493	$2f_{C,2}$
$f_{C,6}$	413	$2f_{C,2}+f_{C,1}$
$f_{C,7}$	357	$2f_{C,2}+2f_{C,1}$
$f_{C,8}$	229	primary
$f_{C,9}$	208	$f_{C,8}+f_{C,1}$

4. MODULATION

Given two functions

$$\alpha(\omega_1, t, \phi_1) = A \sin(\omega_1 \tau + \phi_1)$$

and

$$\beta(\omega_2, t, \phi_2 = B \sin(\omega_1 t + \phi_1)$$

a nonlinear operation R jointly upon α and β yields a function that is Taylor expanded into a series of products of α and β to increasing powers. Exponentiation of a periodic signal merely changes the amplitudes of its harmonics. As each term in R is a product it can be reduced to a series of amplitude modulations and variations in harmonic structure, but the symmetry properties of simple amplitude are no longer applicable. However, the resulting spectrum may still contain features that are linear combinations of the original two fundamental frequencies.

In the unified spectrum of table 1 several lines exhibit spacing that is within the statistical uncertainty of other major features. Lines $f_{C,5}$ and $f_{C,6}$ are separated by *ca.* 2300 years suggesting modulation by the 2300-year line of a higher-frequency carrier. (We borrow the term 'carrier' from communication engineering to denote the higher of two interaction periodic functions; which function is the 'carrier' is a matter of choice.) Moreover, the two lines, $f_{C,8}$ and $f_{C,9}$ are also separated within 2σ by 2300 years, indicating modulation between the 2300-year period and two high-frequency periods. However, the side band expected at 3.93×10^{-3} a^{-1} is not detected.

5. THE RADIOCARBON INVENTORY

Carbon is stored in the atmosphere, oceans, biosphere, and in mineral form. Its accessibility varies greatly; the atmospheric time constant is of order 1–2 years, while at the other extreme at least some of the abyssal oceanic component is recycled in time as great as 200 million years (the maximum age of the ocean floor). Major terrestrial reservoirs of CO_2 are the atmosphere and oceans with a lesser addition from the biosphere. Reservoirs are tabulated by Walker (1977). From the standpoint of geochemical carbon, radiocarbon constitutes a tracer, though ^{14}C clearly is a substance of independent importance when considering its variations and their source(s). Variations in the inventory arise from (1) changes in the incident flux of galactic cosmic rays (interplanetary magnetic field modulation), (2) changes in solar flare associated cosmic rays, (3) modulation of the incoming CR by changes in the Earth's magnetic field, and finally (4) possible variations in the take-up and release of CO_2 by the oceans. The latter is important; later we discuss qualitatively how the long-period spectrum might be forced by oceanic currents.

Of the 10 features identified separately in the La Jolla and Belfast spectra, eight are common to the two (within 2σ). The resulting spectrum of the combined data confirms the earlier statistically tentative surmise that the lines in the 200-year neighbourhood are primary features modulated by the 2300-year line. Because $f_{C,8}$ and $f_{Be,8}$ are common features, it is unlikely that they arise from terrestrial forcing as no evidence exists for geomagnetic variations of the required magnitude. The magnetic record is of insufficient length to clearly rule out magnetic forcing, but neither paleomagnetic or present epoch geomagnetic data provide any evidence (Cox 1969). Chemical forcing is ruled out for Be. The conclusion that *ca.* 200-year periods are extraterrestrial in origin together with the evidence for modulation leads to the likelihood that the 2300-year period is due to terrestrial forcing of the radiocarbon inventory. If radiocarbon is regarded as a tracer for carbon generally, then the global inventory of carbon itself is being forced at 2300 years.

Though $f_{C,2}$ (960 years) does not appear in the beryllium spectrum, the second harmonic ($f_{C,5}$) does. As the former is the weaker of the two it suggests that $f_{Be,2}$ may exist but obscured by noise in the beryllium spectrum. A geomagnetic origin cannot be tested for as the period is in excess of the geomagnetic record. However, there are no linear side band combinations between $f_{C,2}$ and $f_{C,8}$, implying that the two lines do not interact. This is contrary to what would be expected if $f_{C,2}$ were the result of geomagnetic forcing.

6. The joint air temperature $\Delta^{14}C$ correlation

Evidence for a correlation exists between the radiocarbon spectrum and the spectrum of variations in the growth of bristlecone pine trees from Campito Mt in the White Mts of eastern California. Campito Mt bristlecone (from the same general region as the trees which supplied the wood for the La Jolla radiocarbon dating) supplies ring growth information that in the arid environment near timberline, is sensitive to air temperature (La Marche 1973). Sonett & Suess (1984) have shown a close similarity between the autospectra of the La Jolla radiocarbon and growth of these trees. This relation appears also by the cross-MEM spectral, coherence, and phase shown in figure 4. The radiocarbon and tree ring data correlation peaks with a relative shift in lag favouring the tree ring data, i.e. the tree response leads by *ca.* 80 years. Furthermore, the two records are inversely correlated in the sense that thicker tree rings (more rapid growth), if associated with higher temperature and greater solar activity also infers greater interplanetary modulation and thus lessened production of radiocarbon. These are surmises and depend upon the idea that solar activity is positively correlated with atmospheric temperature. The lag is consistent with an expected fast tree response to temperature against a longer atmospheric response to the radiocarbon source.

7. Feedback and oscillators

Line $f_{C,1}$ has been surmised to be associated with variations in the higher-order (non-dipolar) component of the Earth's magnetic field because an apparent cyclicity exists in paleomagnetic directions (Lund 1983; Creer 1983). However, globally averaged radiocarbon and ^{10}Be production rates are unlikely to be sensitive to changes in field direction alone. Moreover, the

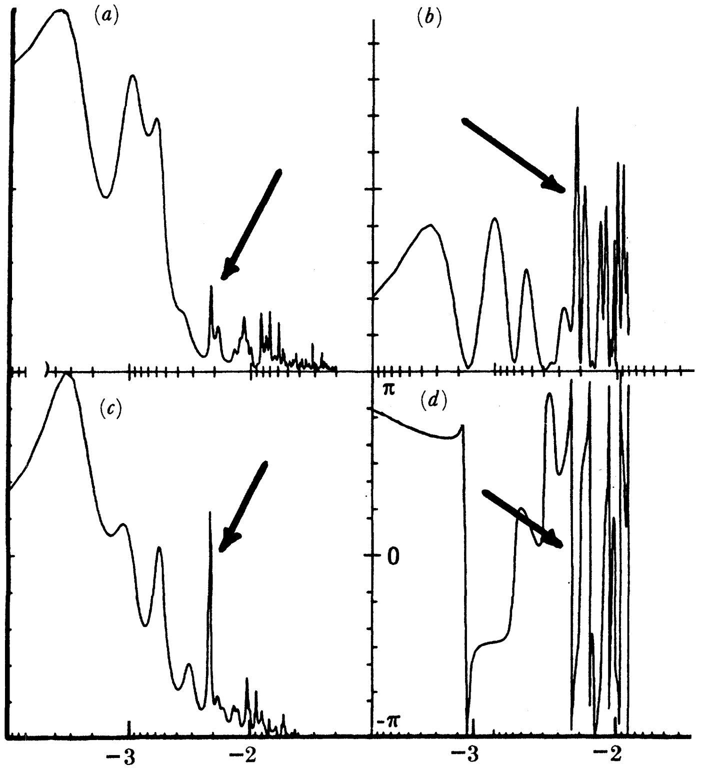

FIGURE 4. MEM cross spectrum of Campito Mt bristlecone pine growth ring sequence and La Jolla $\Delta^{14}C$ sequence. (*a*) Campito autospectrum, (*b*) coherence, (*c*) La Jolla autospectrum, (*d*) relative phase. (Modulo 2π; MEM order = 75, arrows point to a *ca.* 200-year line.)

^{10}Be record lacks a *ca.* 2000-year spectral feature. Because a vague suggestion exists that the Maunder-like minima are amplitude modulated with a period of *ca.* 2000 years a solar origin cannot be ruled out with confidence. Nevertheless, as the ^{10}Be record shows a continuous *ca.* 200-year period and the longer *ca.* 2000-year) period is absent in this record, the argument for a terrestrial source for the *ca.* 2300-year period is strengthened. By a process of elimination, the major candidate appears to be a 'chemical' oscillation.

The tree ring record (aside from possible arcane photosynthesis effects) involves an air-temperature–radiocarbon-production relation at several periods within the radiocarbon spectral range. Can oscillatory oceanic forcing exist and if so can periods be inferred within the radiocarbon range but independent of the ^{10}Be record? It is known that the global deep water cycle times vary from fractional millennia to 1000–2000 years. Such a return path in effect stores carbon except for vertical diffusion and sedimentation (Broecker & Li 1970). If an increase of atmospheric carbon is destablizing with respect to respiration of CO_2 from surface waters, then the deep water return path could function as a kind of delay system. The electrical analogy is a dispersive delay line that for certain periods depending upon circuit constants will shift phase sufficiently to sustain oscillations. Under such conditions the deep-water radiocarbon record becomes a tracer for a global oscillation of that part of the carbon not locked into the geological reservoir. A fanciful example of the return paths is given by Broecker & Peng (1982) from which figure 5 is adapted. The importance of this to climate would rest

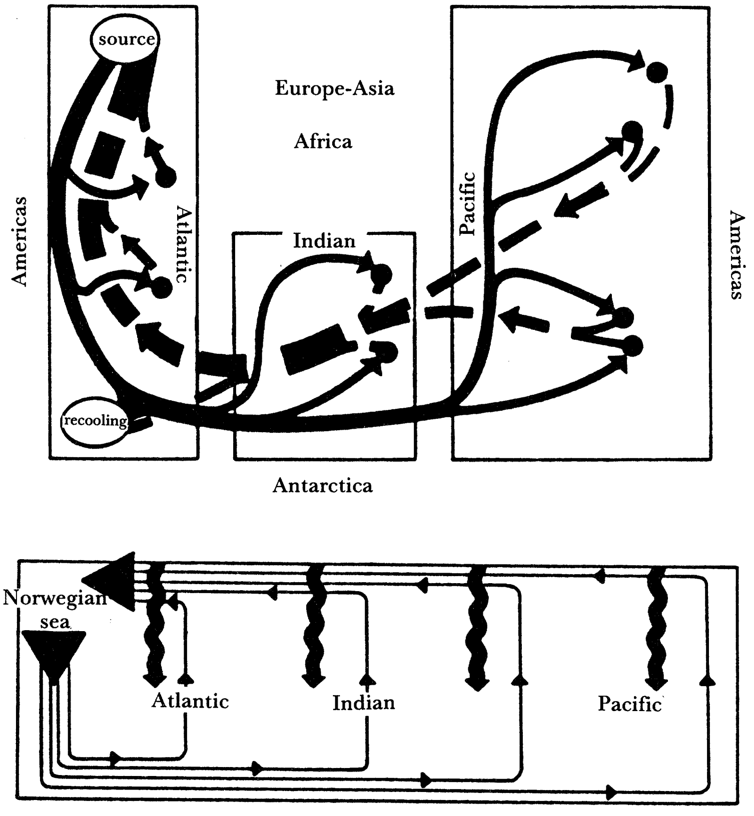

FIGURE 5. A fanciful schematic of the deep water return paths for the world ocean. Return circuit times have been variously reported from 700 to 2000 years based upon deep water radiocarbon ages. Return path corresponds to a frequency sensitive (dispersive) delay line. Combined with positive gain and sufficient phase shift, self-excitation of the system is conjectured. The heavy wave lines represent particle sedimentation, which would have the effect of adding younger radiocarbon, thus decreasing inferred deep water return times. (From Broecker & Peng (1982).)

upon the amplitude of the atmospheric reservoir oscillations at 2300 years that were linked to the oceanic system.

That the Sun can support oscillations reflected in the terrestrial record must rest basically upon solar hydromagnetics for the transit time for waves across the Sun is not more at most of order 1 h. The long terrestrial record is from radiocarbon, but shorter records, still long relative to the transit time come from auroras (Schröder 1988) and from study of sunspots. The Gleissberg period (*ca.* 80–90 years) is obvious in the sunspot index autocorrelation (figure 6). However, reservoir attenuation of the atmosphere spectral line amplitudes of radiocarbon decrease by large factors (*ca.* 200 for 11-year period) (Houtermans, *et al.* 1973). (See also Castagnoli *et al.* 1984.)

Other periods, e.g. 100–150 years, are also present in the radiocarbon record from time to time (de Jong & Mook 1980), but may be non-stationary (Damon *et al.* 1990). Stuiver & Quay (1980) have shown that variability in the radiocarbon record extends downward in period through the Maunder, Spörer, etc., minima (figure 7), whereas Kocharev (1987) has shown that the variability seems to exist even in the 11-year cycle (see also Beer *et al.* 1983). Kocharev's $\Delta^{14}C$ record furthermore implies the continuation of solar activity through the

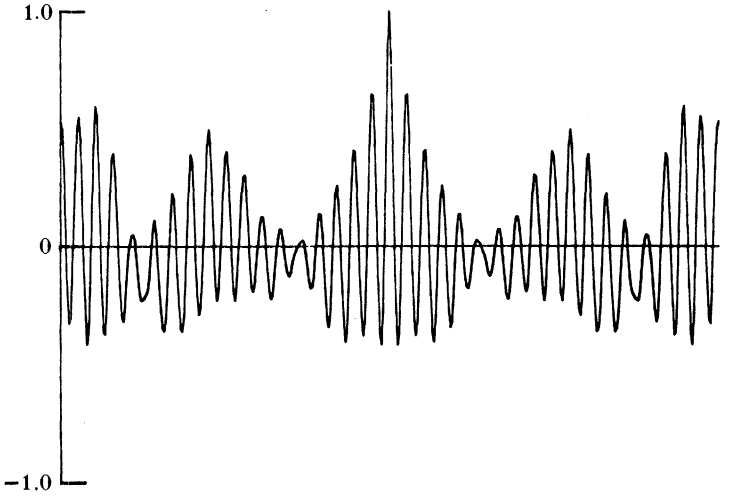

FIGURE 6. Autocorrelation of Wolf sunspot index numbers, showing prominent 80–90-year amplitude modulation (Gleissberg period) of the basic 11-year solar activity cycle.

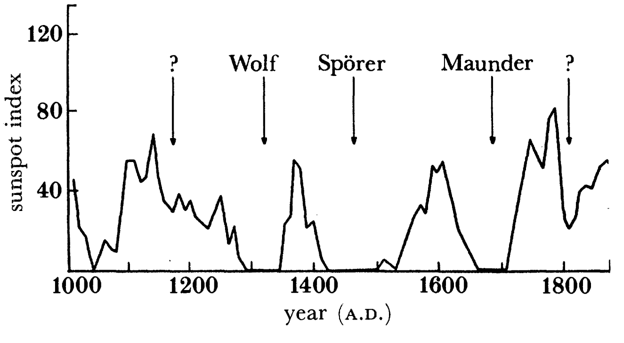

FIGURE 7. Reconstruction of the sunspot index for the past millennium from Pacific northwest radiocarbon (From Stuiver & Quay (1980).)

Maunder Minimum with a change in the primary period from 11 to 22 years during the interval!

The combination of a small fossil magnetic field in the solar core added to the dynamo field, all modulated by a field having the Gleissberg period, yields a spectrum similar to the observed sunspot index spectrum (Sonett 1984). Substitution in this model of a very-long-period field for the fossil (or in addition to it) may provide power in the frequency range of interest for the radiocarbon spectrum. In the Sonett model the substitution is made for the fossil field so that now the sunspot index has the general form (restricted to amplitude modulation) of

$$S = [(1+\alpha \cos \omega_{m1} t)\,(\cos \omega_c t + \cos \omega_{m2} t)]^2 + n(t)^2, \tag{3}$$

where α is the amplitude modulation factor, ω_{m1} the long-period modulation frequency, ω_c the Hale period, ω_{m2} an additional longer period, and $n(t)$ noise. Such a model is *ad hoc* in the sense of the addition of ω_{m2} and a physical model must be developed to support this conjecture.

The major evidence that the 200-year period is solar comes from the suspected modulation by the longer *ca.* 2300-year period, inferring a terrestrial modulation of a periodic

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extraterrestrial source, but even this is not unique. The evidence that line features are non-stationary in amplitude and perhaps even in frequency increases the difficulty in making assignments of sources. It should be noted that the general shape of the radiocarbon spectrum is red, i.e. increasing power with period. Together with the transient stability of the spectra (Sonett 1984) the suggestion is arguable that the underlying dynamical system responsible for the variations is chaotic or at least quasi-period (as seen in the time period discussed in this paper) (see, for example, Thompson & Steward 1982). However, as noted earlier the low-pass filter properties of the atmosphere may explain the 'redness'. Moreover, tests for fractional dimensionality require, at the least, evenly spaced data (Grassberger & Procaccia 1983), which is not satisfied by the La Jolla sequence. Although the Belfast sequence is nearly so, its length is only half that of the La Jolla sequence that already, in view of its low signal:noise ratio, is marginal for fractional dimensionality tests. Lastly, Feynman & Gabriel (1990) have studied evidence that the Gleissberg cycle and Maunder minima are tied together through quasi-periodicity or chaotic behaviour. Thus this matter remains unsettled.

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8. References

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Discussion

A. Berger (*Département de Physique*, *Université Catholoque de Louvain*, *Belgium*). Professor Sonett's discovery of a 2300-year cycle is very interesting. Such a quasi-period has also recently been found in ^{18}O deep sea cores with high sedimentation rates (Pestiaux *et al.* 1987). This was tentatively related to the ocean circulation and the astronomical theory. Can Professor Sonett comment on a possible relation between what causes this 2300-year period in both his data and in mine.

C. P. Sonett. Evidence for a spectral feature in the neighbourhood of 2300–2500 years in various natural time series ranges from the ^{18}O record through radiocarbon, and the fascinating sea core data reported by Pestiaux *et al.* I think it is far too early to attempt to assign an overall mechanism, but as is well known the oxygen record is related to rainfall. Some evidence for this period in the bristlecone pine record also appears. If real it indicts temperature as these trees are at timberline and only thermally stressed (C. W. Stockton, personal communication). That the period also appears in the radiocarbon record is especially difficult to understand because it could be due to solar wind modulation, perhaps deep ocean circulation, or even the Earth's magnetic field which displays a directional 'coning' (precession) of 2400-year period (though I doubt whether this could affect the radiocarbon inventory.

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